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Little hierarchy, little higgses, and a little symmetry

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Abstract: Little Higgs theories are an attempt to address the "little hierarchy problem," i.e., the tension between the naturalness of the electroweak scale and the precision electroweak measurements showing no evidence for new physics up to 5 – 10 TeV. In little Higgs theories, the Higgs mass-squareds are protected at one-loop order from the quadratic divergences. This allows the cutoff of the theory to be raised up to ~ 10 TeV, beyond the scales probed by the current precision data. However, strong constraints can still arise from the contributions of the new TeV scale particles which cancel the one-loop quadratic divergences from the standard model fields, and hence re-introduces the fine-tuning problem. In this paper we show that a new symmetry, denoted as T-parity, under which all heavy gauge bosons and scalar triplets are odd, can remove all the tree-level contributions to the electroweak observables and therefore makes the little Higgs theories completely natural. The T-parity can be manifestly implemented in a majority of little Higgs models by following the most general construction of the low energy effective theory à la Callan, Coleman, Wess and Zumino. In particular, we discuss in detail how to implement the T-parity in the littlest Higgs model based on SU(5)/SO(5). The symmetry breaking scale f can be even lower than 500 GeV if the contributions from the higher dimensional operators due to the unknown UV physics at the cutoff are somewhat small. The existence of T-parity has drastic impacts on the phenomenology of the little Higgs theories. The T-odd particles need to be pair-produced and will cascade down to the lightest T-odd particle (LTP) which is stable. A neutral LTP gives rise to missing energy signals at the colliders which can mimic supersymmetry. It can also serve as a good dark matter candidate.

Keywords: Beyond Standard Model, Phenomenological Models.

JHEP08(2004)061

Contents

1. Introduction

The origin of the electroweak symmetry breaking, a fundamental mystery of the weak interactions, will be probed directly by high energy experiments in the coming decade. In the standard model (SM) the symmetry breaking is triggered by the vacuum expectation value (VEV) of a scalar Higgs field. The quantum correction to the mass of the Higgs particle is, however, very sensitive to the ultraviolet (UV) physics. To produce the observed electroweak breaking scale, the radiative corrections to the Higgs mass-squared need to be fine-tuned, unless new particles are introduced at around the 1 TeV scale to cut off the quadratically divergent contributions. Generally speaking, if the scale of the electroweak symmetry breaking is to be stabilized naturally, we expect new physics to show up at the scale ~ 1 TeV or below.

At energies below the scale of new physics, new particles can be integrated out to obtain a set of higher dimensional operators involving the standard model fields only [1]. These higher dimensional operators can contribute to experimental observables even at energies beneath the scales of new physics, and the size of these operators are constrained by precision measurements [2]. The strongest bounds are on the operators violating the (approximate) symmetries of the standard model, such as baryon number, flavor and CP symmetries. The dimensionful parameters suppressing these higher dimensional operators typically need to be above 100 TeV due to the absence or rareness of events which violate these symmetries. This implies that the new physics at the TeV scale should also respect these symmetries (at least approximately). On the other hand, operators preserving the symmetries of the standard model are expected to be induced by new physics at the TeV scale. These operators can be probed by the electroweak precision measurements. The advances of the electroweak precision measurements in the past decade put significant constraints on many of these operators. The most constrained operators need to be suppressed by the energy scales $5 - 10\,\mathrm{TeV}$ [3], assuming that all dimensionless coefficients are $\mathcal{O}(1)$. Taken at face value, it seems to indicate that there is no new physics up to $\sim 10\,\mathrm{TeV}$, which creates a tension, known as the little hierarchy problem, with the naturalness requirement that new physics should appear at $\sim 1\,\mathrm{TeV}$ or below in order for the electroweak symmetry breaking to be stabilized. This discrepancy might be accidental and the electroweak scale is fine-tuned. However, it is also quite possible that the little hierarchy problem is in fact giving us an important hint of the new physics at the TeV scale and ought to be taken more seriously.

In the supersymmetric extension of the standard model, a conserved R-parity can be imposed to eliminate all the tree-level contributions to the electroweak observables from the superpartners so that the coefficients of the higher dimensional operators are naturally suppressed by a loop factor. This would relieve the little hierarchy problem. However, there is still a related fine-tuning problem in supersymmetric theories. In the standard model the Higgs particle has a special status as the only scalar in the theory. In supersymmetric theories there are, nevertheless, many superpartners which are also scalars. Generically superpartner masses are expected to be in the same order as the Higgs mass since the Higgs is not special. However, the loop contributions to the electroweak observables from some superpartners as light as $\mathcal{O}(100\,\mathrm{GeV})$ could still be dangerous (e.g., $\Delta\rho$ from top squark loops). Moreover, in the minimal supersymmetric standard model (MSSM), raising the lightest Higgs mass above the experimental bound requires large top squark masses. On the other hand, the radiative corrections to the Higgs mass from the top squark masses are at least as large as the top squark masses themselves if one requires the theory to be valid up to the energy scale of the gauge coupling unification. As a result, such heavy top squarks, though more phenomenologically desirable from above discussion, introduce fine-tuning at the level of a few percent or less. This naturalness problem in supersymmetry has been discussed in many places before [4]–[8]. The fact that so far we have found neither the Higgs, the superpartners, nor deviations from the electroweak precision data, indicates that the electroweak scale still appears to be somewhat fine-tuned for the simplest supersymmetric extensions of the standard model, even if supersymmetry is the answer to the physics beyond the standard model.

Little Higgs theories [9]–[18] provide an alternative approach to stabilize the electroweak scale based on a non-linear chiral lagrangian of a coset space G/H. The Higgs is light because it is a pseudo-Nambu-Goldstone boson (PNGB) with the unique property that its mass is protected at one-loop order from the quadratic divergences. The cutoff can therefore be raised above $\sim 10\,\mathrm{TeV}$, beyond the scales probed by the current electroweak precision measurements. Consequently the contributions from the UV physics above the cutoff are in general safe from the electroweak constraints, as long as the unbroken group H contains a custodial SU(2) symmetry. The one-loop quadratically divergent contributions to the Higgs mass from the SM top quark, gauge fields and Higgs itself are cancelled by contributions from new particles at the $\sim 1\,\mathrm{TeV}$ scale with the same spins as the SM particles. In contrast to supersymmetry, there is a natural separation in the mass scale of the Higgs and the new TeV particles, since the Higgs is the only PNGB whose mass is suppressed by a two-loop factor.

The electroweak observables also receive contributions from the new TeV particles in little Higgs theories, and there have been several studies on this issue for various little Higgs models [19]–[27]. The results are quite model and parameter dependent, although the general impression is that the electroweak precision data impose strong constraints on the symmetry breaking scale of G in most of the parameter space, and hence the TeV particles which cancel the quadratic divergences are required to be heavy. This in turn re-introduces fine-tuning into the little Higgs models. A closer look shows that the strongest constraints come from the direct couplings of the SM fields to the new gauge bosons, as well as the VEV of any SU(2) scalar triplet which arises from coupling to the SM Higgs. These couplings, however, are not an essential part of the little Higgs theories; they do not participate in the cancellation of the quadratic divergences of the Higgs mass corrections. If there is a natural way, e.g., by imposing a new symmetry, to suppress these dangerous couplings in a little Higgs model, it would be completely consistent with the precision electroweak constraints without any fine tuning. Indeed this was achieved in ref. [28] for a three-site moose model, where a T-parity forbids all these dangerous couplings.

In this paper, we discuss how the dangerous couplings in little Higgs theories can be removed in a more general context. The T-parity is usually easy to be incorporated into the scalar and the gauge sectors. However, it was not respected by the fermion sectors in the conventional little Higgs models. This is due to the fact that in the original models and the follow-up phenomenological studies, the SM fermions are assigned to particular representations under the broken group G. From a low energy effective theory point of view, however, only the symmetry transformation property of a particle under H is unambiguous since only the unbroken group H is manifest. As Callan, Coleman, Wess and Zumino [29, 30] (CCWZ) showed long time ago, by starting with a lagrangian manifestly invariant under the unbroken group H and matter content furnishing linear representations of H, one can construct a lagrangian non-linearly realizing the full symmetry G with the help of Nambu-Goldstone fields which parametrize the coset space G/H. From the low energy effective theory perspective, this is a more appropriate way to construct the effective lagrangian and it is not necessary to introduce linear representations of the full group G at all. The advantage of this approach in our context is that there is a natural separation of the model-dependent dangerous interactions from those essential interactions which engineer the little Higgs mechanism. In a majority of the little Higgs models, one can easily identify a little symmetry—the T-parity, which can be imposed to remove all the unwanted dangerous couplings, rendering these models completely natural.

In the next section, we start with the minimal moose model of ref. [11]. It is an ideal example for the illustrative purpose because of its similarity to the very familiar QCD chiral lagrangian. In section 3 we move on to the most popular SU(5)/ SO(5) littlest Higgs model [12] and show how the T-parity can be implemented to remove the strong constraints from the electroweak precision data. The existence of the T-parity has a drastic impact on the little Higgs phenomenology, which will be briefly discussed in section 4. We will comment on general models in which T-parity can or cannot be naturally implemented and draw the conclusions in section 5.

2. An SU(3) minimal moose model with T parity

We start our discussion with the minimal moose model [11], a two-site model based on SU(3) global symmetry. This model does not have a custodial SU(2) symmetry, and a large contribution to the $\Delta\rho$ parameter is already present in the non-linear sigma model, which is independent of the contributions from the TeV scale particles. However, this can be easily cured by replacing the SU(3) group with another group, such as SO(5), containing a custodial SU(2). Despite of the strong electroweak constraints on this model, we will nevertheless use it to illustrate our main point because the model shares the same global symmetry breaking pattern, $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \to \mathrm{SU}(3)_V$, with the very familiar QCD chiral lagrangian. The purpose here is to show how a T-parity can be naturally imposed to get rid of the dangerous tree-level couplings between SM fermions and the heavy gauge bosons.

The QCD chiral lagrangian has a well-known Z_2 symmetry: the parity which exchanges the chirality $L \leftrightarrow R$. Baryons, transforming as octets under the unbroken $\mathrm{SU}(3)_V$, can be incorporated in the chiral lagrangian in many different ways. The freedom lies in the assignments of the full $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ representations under which baryons transform, since there are many representations containing an octet of the unbroken $\mathrm{SU}(3)_V$ after chiral symmetry breaking. Not every assignment respects the parity symmetry in a manifest way. Alternatively, we could also not worry about how baryons transform under the full chiral group and contend ourselves with the fact that baryons form an octet under the unbroken group à la CCWZ. In any case, parity is a good symmetry of QCD and can be imposed in the chiral lagrangian [31, 32]; the parity transformation is non-linear and involves the Goldstone bosons if baryons are assigned to a parity non-invariant representation of $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$.

We run into a completely similar situation in the minimal moose model. There is an incarnation of the parity in QCD: the Z_2 reflection which interchanges the two sites. However, a naïve assignment of the SM fermion under the full symmetry group does not respect the reflection symmetry, resulting in direct couplings between the standard model fermions and the heavy gauge bosons. Nevertheless, as in the QCD chiral lagrangian, a linear Z_2 parity can be consistently implemented to forbid these unwanted couplings. The parallel problem in the QCD is how to write down a chiral lagrangian including fermions which linearly realizes the parity. This problem has been explicitly worked out in ref. [31, 32], which is an adaption of CCWZ to the chiral symmetry breaking. We will see that, by following this approach, a T-parity which removes the electroweak constraints from the new TeV particles in the minimal moose model will become manifest.

The minimal moose model is a two-site SU(3) model with four links $X_i = \exp(2ix_i/f)$, $i = 1, \ldots, 4$. We gauge the same $\mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup within both SU(3) groups with equal strength.[1] The Goldstone boson matrices, x_i, each contains a triplet, a doublet and a singlet under the unbroken diagonal $\mathrm{SU}(2)_V$ gauge group, which we take to be the electroweak $\mathrm{SU}(2)_W$. This model has a large global symmetry $[\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R]^4$

[1] This is slightly different from the original model which gauged an SU(3) on one site and $\mathrm{SU}(2) \times \mathrm{U}(1)$ on the other. The reason is we want the gauge sector to respect the Z_2 reflection symmetry. This minor difference does not affect the little Higgs mechanism.

JHEP08(2004)061

spontaneously broken down to $[\mathrm{SU}(3)_V]^4$, with the non-linear realization

$$X_i \to L_i X_i R_i^\dagger, \quad i = 1, \ldots, 4\,. \tag{2.1}$$

The effective lagrangian is written as

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_X + \mathcal{L}_\psi + \mathcal{L}_t\,. \tag{2.2}$$

Here $\mathcal{L}_G$ contains the conventional non-linear sigma model kinetic terms and gauge interactions, whereas $\mathcal{L}_X$ includes the plaquette operators which give rise to Higgs quartic interactions:

$$\mathcal{L}_X = \kappa f^4 \left[\mathrm{Tr}(X_1 X_2^\dagger X_3 X_4^\dagger) + \mathrm{Tr}(X_2 X_3^\dagger X_4 X_1^\dagger)\right] + \text{h.c.}\,, \tag{2.3}$$

where $f \sim 1\,\mathrm{TeV}$ sets the cutoff of the theory to be at $\Lambda \sim 4\pi f \sim 10\,\mathrm{TeV}$. The gauge and scalar sectors obviously respect the parity,

$$A_L \leftrightarrow A_R, \;\; \text{where } A_{L,R} \text{ are gauge fields for } [\mathrm{SU}(2)\times\mathrm{U}(1)]_{L,R}, \tag{2.4}$$

$$X_i \leftrightarrow X_i^\dagger\,, \tag{2.5}$$

under which the SM gauge bosons are even, and all the Goldstone bosons and heavy gauge bosons are odd. In order to make the standard model Higgs even under the parity, we note that the matrix $\Omega_X = \mathrm{diag}(1, 1, -1)$ commutes with all the gauge generators and can be included in the parity transformation to flip the parity of the Higgs doublet without affecting anything else. (This is in fact equivalent to a hypercharge rotation.) We therefore define the T-parity for the gauge and the scalar sectors as,

$$A_L \leftrightarrow A_R\,, \tag{2.6}$$

$$X_i \leftrightarrow \Omega_X X_i^\dagger \Omega_X\,. \tag{2.7}$$

Under the T-parity, all SM gauge bosons and Higgs doublets are even, and the heavy gauge bosons and scalar triplets and singlets are odd. One can immediately see that with the T-parity, there is no mixing between the SM gauge bosons and the heavy gauge bosons, and the coupling of the scalar triplets to two Higgs doublets is forbidden. So there is no tadpole term to induce the triplet VEVs after electroweak symmetry breaking.

In the fermionic sector $\mathcal{L}_\psi$, the right-handed (electroweak singlet) fermions can be easily incorporated to respect the T-parity: we assign them to be charged only under the diagonal $\mathrm{U}(1)_V$, (i.e., they carry equal charges under $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_R$,)

$$\mathcal{L}_\psi \supset \overline{\psi}_S\, \sigma^\mu (\partial_\mu + i g_Y Y_{\psi_S} B_\mu)\, \psi_S = \overline{\psi}_S\, \sigma^\mu \left(\partial_\mu + i\sqrt{2} g_Y \frac{Y_{\psi_S}}{2}(B_{L\mu} + B_{R\mu})\right) \psi_S\,, \tag{2.8}$$

where $\sigma^\mu = (1, \vec{\sigma})$ and B_μ is the SM hypercharge gauge field which is a linear combination of the $\mathrm{U}(1)_L$ gauge field $B_{L\mu}$ and $\mathrm{U}(1)_R$ gauge field $B_{R\mu}$. To realize the T-parity linearly for the left-handed (electroweak doublet) fermions, we define the matrix $\xi = \exp(ix/f)$, so that ξ is the square root of the link field, $\xi^2 = X$ [31, 32]. Under the global $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ transformations,

$$\xi \to \xi' = L \xi U^\dagger = U \xi R^\dagger\,, \tag{2.9}$$

where U belongs to the unbroken $\mathrm{SU}(3)_V$ and is a non-linear function of L, R, and the Goldstone bosons x. Now for each electroweak doublet fermion in the standard model, ψ_D, we take it to transform under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ as

$$\psi = \begin{pmatrix} \psi_D \\ 0 \end{pmatrix} \to U\psi \,. \tag{2.10}$$

If ψ were a complete multiplet of the unbroken $\mathrm{SU}(3)_V$, eq. (2.10) would define a a non-linear representation of the $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ group. To write down the kinetic term for the fermions, we need to compute the Maurer-Cartan one form [29, 30]

$$\xi^\dagger D_\mu \xi \equiv \xi^\dagger(\partial_\mu + iA^a_\mu Q^a_V + i\mathcal{A}^a_\mu Q^a_A)\xi \equiv v^a_\mu T^a + p^a_\mu X^a \,, \tag{2.11}$$

where $A^a_\mu, \mathcal{A}^a_\mu$ are the unbroken and broken gauge fields, respectively, and T^a, X^a are the unbroken and broken generators. The Maurer-Cartan one form takes value in the Lie algebra of the full symmetry group G, therefore it can be written as linear combinations of T^a and X^a. The components of $\xi^\dagger D_\mu \xi$ in the direction of unbroken and broken generators are defined as v^a_μ and p^a_μ respectively. Under the U rotation, v_μ transforms like a gauge field and p_μ transforms covariantly,

$$v^a_\mu T^a \to U v^a_\mu T^a U^\dagger + U(\partial_\mu U^\dagger) \,, \qquad p^a_\mu X^a \to U p^a_\mu X^a U^\dagger \,. \tag{2.12}$$

These objects can be used to write down lagrangians invariant under the full broken group G. For example, a fermion kinetic term is given by

$$\overline{\psi}\, \bar{\sigma}^\mu \left(\partial_\mu + v^a_\mu T^a + i\, g_Y (Y_{\psi_D} - Y_H) B_\mu\right) \psi \,, \tag{2.13}$$

where $\bar{\sigma}^\mu = (1, -\vec{\sigma})$. Apart from the gauge interactions it realizes the full $\mathrm{SU}(3) \times \mathrm{SU}(3)$ non-linearly. The additional term involving B_μ serves to give the correct hypercharge to the SM fermion ψ_D.

There is a compact way to write down $v^a_\mu T^a$ by noting that, under the parity which interchanges $L \leftrightarrow R$, the generators $T^a \to T^a$ and $X^a \to -X^a$. Then eq. (2.11) turns into

$$\xi D_\mu \xi^\dagger = \xi(\partial_\mu + iA^a_\mu Q^a_V - i\mathcal{A}^a_\mu Q^a_A)\xi^\dagger = v^a_\mu T^a - p^a_\mu X^a \,. \tag{2.14}$$

The fermion kinetic term, eq. (2.13), can now be written as

$$\overline{\psi}\, \bar{\sigma}^\mu \left(\partial_\mu + \frac{1}{2}(\xi_4^\dagger D_\mu \xi_4 + \xi_4 D_\mu \xi_4^\dagger) + i\, g_Y (Y_{\psi_D} - Y_H) B_\mu\right) \psi, \tag{2.15}$$

where, to be more concrete, we have inserted ξ_4, the square root of the fourth link X_4, although it could be any of the links. Eq. (2.15) is manifestly invariant under T-parity, since the SM fermions are taken to be T-even. By expanding v^a_μ, one can check that at lowest order (with no Goldstone field), it only contains the standard model gauge interactions for the fermions, but not the direct couplings between the fermions and the heavy gauge bosons $\mathcal{A}_\mu$.

JHEP08(2004)061

Another term invariant under the full $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry which we can write down is

$$g_A \overline{\psi}\, \bar{\sigma}^\mu p^a_\mu X^a \psi = g_A \overline{\psi}\, \bar{\sigma}^\mu (\xi^\dagger D_\mu \xi - \xi D_\mu \xi^\dagger)\psi\,, \tag{2.16}$$

with an *arbitrary* coefficient g_A. It contains the dangerous direct couplings between the SM fermions and the heavy gauge bosons, which is the origin of the strong electroweak precision constraints on the little Higgs models. However, one can easily see that this term is T-odd, since the T-parity interchanges $\xi^\dagger D_\mu \xi$ and $\xi D_\mu \xi^\dagger$ in eq. (2.16),[2] and can be forbidden by imposing the T-parity. The nice thing about following the approach of ref. [29, 30, 31, 32] is that the essential T-preserving interactions and the model-dependent T-violating interactions are naturally separated, thus allowing for an easy identification of the T-parity which removes the strong electroweak constraints on the little Higgs models.

In terms of gauge fields A_μ and $\mathcal{A}_\mu$, Goldstone fields x_i, and the fermion ψ, the T-parity defined as

$$T : A_\mu \to A_\mu\,, \qquad \mathcal{A}_\mu \to -\mathcal{A}_\mu\,, \qquad x_i \to -\Omega_X x_i \Omega_X\,, \qquad \psi \to \Omega_X \psi \tag{2.17}$$

is an exact symmetry of $\mathcal{L}_G + \mathcal{L}_X + \mathcal{L}_\psi$. The remaining task is to write down the standard model Yukawa coupling $\mathcal{L}_t$, which was worked out in ref. [11], except that we need to T-symmetrize the Yukawa interactions. For the top Yukawa coupling we introduce a vector-like pair of T-even, colored Weyl fermions u' and u'^c:

$$\mathcal{L}_t \supset \frac{\lambda}{2} f (b\ t\ u') \left[\xi_1 \xi_2^\dagger + \Omega_X \xi_1^\dagger \xi_2 \Omega_X\right] \begin{pmatrix} 0 \\ 0 \\ u_3^c \end{pmatrix} + \lambda' f u'^c u', \tag{2.18}$$

so that the quadratically divergent contribution to the Higgs mass-squared induced by the large top Yukawa coupling is cancelled by the additional fermion u'. For the light up-type quarks, the Yukawa coupling has the same form as eq. (2.18) except that u' and u'^c are not needed, whereas for the down-type quarks and charged leptons, the Yukawa coupling can be written as

$$\mathcal{L}_t \supset \frac{\lambda_d}{2} f (0\ 0\ d^c) \left[\xi_1 \xi_2^\dagger + \Omega_X \xi_1^\dagger \xi_2 \Omega_X\right] \begin{pmatrix} q \\ 0 \end{pmatrix} + \frac{\lambda_e}{2} f (0\ 0\ e^c) \left[\xi_1 \xi_2^\dagger + \Omega_X \xi_1^\dagger \xi_2 \Omega_X\right] \begin{pmatrix} l \\ 0 \end{pmatrix}. \tag{2.19}$$

With these Yukawa interactions, the T-parity defined in eq. (2.17) is also an exact symmetry of the Yukawa sector $\mathcal{L}_t$, and hence a symmetry of the full effective lagrangian $\mathcal{L}$.

The matter content of the T-invariant SU(3) minimal moose model is very similar to the original minimal moose model in ref. [11]. Because the gauge group is $[\mathrm{SU}(2) \times \mathrm{U}(1)]^2$, only two scalar triplets and singlets are eaten through the Higgs mechanism after symmetry breaking. Naïvely it seems that in the T-invariant model there are three doublets remaining light since only one becomes heavy due to the plaquette operators eq. (2.3). An additional doublet would have been eaten by the heavy gauge bosons if the gauge group were $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ as in the original minimal moose model. However, the fermions

[2] Note that $\Omega_X Q^a_{A,V} \Omega_X = Q^a_{A,V}$, and $\Omega_X \psi = \psi$ since ψ only has the upper two components.

JHEP08(2004)061

in eq. (2.10) do not form a complete multiplet of the global symmetry. The kinetic term eq. (2.15) breaks all chiral symmetries associated with ξ_4. The scalar doublet residing in ξ_4 is no longer protected and picks up a mass of the order of $gf \sim 1\,\mathrm{TeV}$. Diagrammatically, a two loop *quartic-divergent* contribution to the doublet mass-squared is indeed generated through the two-loop diagram in figure 1,[3]

$$m_h^2 \sim \frac{g^2}{(16\pi^2)^2}\frac{\Lambda^4}{f^2} = (gf)^2 \sim (1\,\mathrm{TeV})^2\,. \tag{2.20}$$

Therefore below 1 TeV, the matter content of the T-invariant minimal moose model is simply the two-Higgs-doublet standard model, the same as the original SU(3) minimal moose. At around 1 TeV, there are three sets of scalar triplets and singlets, two doublets, as well as massive SU(2) gauge bosons B', W' and Z', and one vector-like colored fermion responsible for cancelling the quadratic divergence from the top loop.

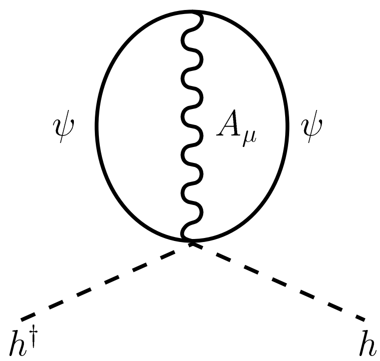

Figure 1: The two-loop diagram with the quartic divergence.

Before moving on to other models, we note that for the minimal moose model the quartic divergence from figure 1, which gives the doublet a heavy mass, is not problematic because there are extra electroweak doublets in the model to spare. For little Higgs models which do not contain any extra electroweak doublet, it would give a contribution to the Higgs mass-squared in the order of $(gf)^2 \sim (1\,\mathrm{TeV})^2$ if the cutoff is $\sim 10\,\mathrm{TeV}$, and hence destabilizing the electroweak scale. It is therefore desirable to see if there is a way to cancel the two-loop quartic divergences. As we discussed above, the divergence arises due to the incomplete fermion multiplet in the kinetic term (2.13) which breaks all chiral symmetries. The remedy is thus to complete the fermion multiplet by introducing a vector-like pair of Weyl fermions χ and χ^c so that the fermion ψ now transforms as a complete fundamental under the $\mathrm{SU}(3)_V$:

$$\psi = \begin{pmatrix} \psi_D \\ \chi \end{pmatrix} \to U\psi\,. \tag{2.21}$$

In this way, under the global $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry, $\xi\psi \to L(\xi\psi)$ transforms like the fermion living on the L-site, while $\xi^\dagger\psi \to R(\xi^\dagger\psi)$ transforms like the fermion living on the R-site. Then each interaction in the kinetic term eq. (2.15) preserves either the $\mathrm{SU}(3)_L$ or the $\mathrm{SU}(3)_R$ which keeps the doublet in ξ_4 from getting a mass. In terms of Feynman diagrams the quartic divergence in figure 1 is cancelled by the diagrams involving the χ fermion and the massive gauge bosons, as shown in figure 2. The fermions χ and χ^c are T-odd because $\psi \to \Omega_X\psi$ under T-parity. The Dirac mass $m_\chi\chi^c\chi$, acting as the cutoff of the divergence in figure 1, needs to be below $\sim 4\pi\sqrt{m_h f/g}$ for naturalness, which implies m_χ only need to be as low as $\sim 3\,\mathrm{TeV}$.

In ref. [28] a three-site model with T-parity was constructed. The gauge couplings on the third site are large and the particles associated with the third site are heavy, being

[3]This contribution was first noted by Nima Arkani-Hamed.

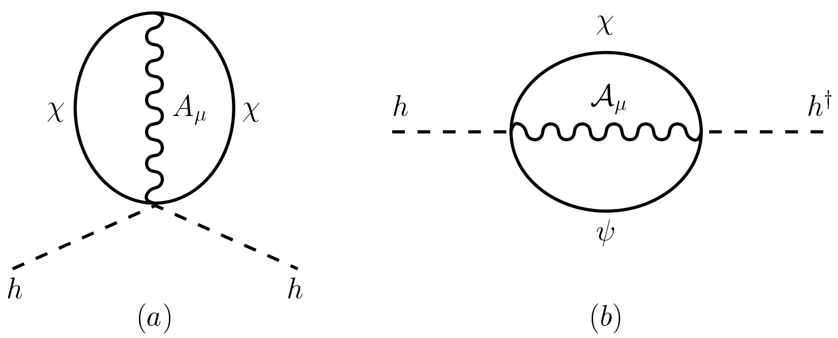

Figure 2: The diagrams responsible for cancelling the quartic divergence. Diagram (a) is there only for the $\mathrm{U}(1)_Y$ gauge boson since χ is an $\mathrm{SU}(2)_W$ singlet and doesn't couple to W and Z bosons directly.

close to the cutoff scale, so it is appropriate to integrate them out below their masses. The low energy effective theory after integrating out the third site is then described by the T-invariant two-site model discussed here, except for the trivial choices of the SU(3) or SO(5) groups. In this way, the three-site model may be viewed as a possible UV extension of the two-site model, in which all fermions transform linearly under the full symmetry, although in principle there could be other possibilities.

3. An SU(5)/ SO(5) model with T parity

Having the experience with the QCD-like minimal moose model, we can now generalize this construction to other little Higgs models. In particular, we will focus on the littlest Higgs model based on an SU(5)/ SO(5) non-linear sigma model [12], which is the most extensively studied little Higgs model in the literature. It has been claimed that, in order to be compatible with the precision data, the original model needs to be fine-tuned and the viable parameter space is thus severely constrained [20, 19, 21]. From the discussion in the previous sections, however, we can see that the strong constraints can be removed completely, hence providing a natural model for electroweak symmetry breaking if the T-parity can be implemented. In this section we show that this is indeed possible. A few more new states are needed to complete the construction, which will be explained along the way.

Following the notation and the basis in ref. [12], we consider a non-linear sigma model arising from a 5×5 symmetric matrix Φ, transforming under the global SU(5) symmetry as $\Phi \to V\Phi V^T$, with a vacuum expectation value

$$\Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix}, \tag{3.1}$$

which breaks SU(5) $\to$ SO(5). The unbroken generators T^a and broken generators X^a satisfy $\Sigma_0 T_a \Sigma_0 = -T_a^T$ and $\Sigma_0 X_a \Sigma_0 = X_a^T$, respectively. Same as $[\mathrm{SU}(3) \times \mathrm{SU}(3)]/\,\mathrm{SU}(3)$, SU(5)/ SO(5) is also a symmetric space in which the unbroken and broken generators satisfy the following schematic commutation relations,

$$[T^a, T^b] \sim T^c\,, \qquad [T^a, X^b] \sim X^c\,, \qquad [X^a, X^b] \sim T^c\,. \tag{3.2}$$

The Lie algebra of a symmetric space therefore has an automorphism: $T^a \to T^a$ and $X^a \to -X^a$, which in the case of SU(5)/ SO(5) can be expressed as $\tau^a \to -\Sigma_0(\tau^a)^T\Sigma_0$ for any generator τ^a. It is this automorphism that allows us to define the T-parity consistently. The Goldstone fields, $\Pi = \pi^a X^a$, are parametrized as

$$\Sigma(x) = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f} = e^{i2\Pi/f}\Sigma_0\,. \tag{3.3}$$

Two different SU(2) $\times$ U(1) subgroups of SU(5) are gauged with equal strength:

$$Q_1^a = \begin{pmatrix} \sigma^a/2 & \\ & \end{pmatrix}, \qquad Y_1 = \mathrm{diag}(3,3,-2,-2,-2)/10\,, \tag{3.4}$$

$$Q_2^a = \begin{pmatrix} & \\ & -\sigma^{a*}/2 \end{pmatrix}, \qquad Y_2 = \mathrm{diag}(2,2,2,-3,-3)/10\,. \tag{3.5}$$

The unbroken gauge group is identified with the electroweak $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$, under which the Goldstone bosons decompose into a doublet, taken to be the Higgs doublet, and a triplet:

$$\Pi = \begin{pmatrix} & \frac{H}{\sqrt{2}} & \phi \\ \frac{H^\dagger}{\sqrt{2}} & & \frac{H^T}{\sqrt{2}} \\ \phi^\dagger & \frac{H^*}{\sqrt{2}} & \end{pmatrix}, \tag{3.6}$$

where we have omitted Goldstone bosons eaten by the Higgs mechanism. In components the Higgs scalars are $H = (h^+\ h^0)^T$. A Higgs VEV, $\langle H\rangle = (0\ v/\sqrt{2})^T$ triggers the electroweak symmetry breaking. The gauge interactions break the global SU(5) symmetry, but they are chosen in such a way that, when the gauge coupling of $[\mathrm{SU}(2) \times \mathrm{U}(1)]_1$ is turned off, there is an enhanced global $\mathrm{SU}(3)_1$ living in the upper-left 3×3 block of the SU(5) group. Conversely, when the coupling of $[\mathrm{SU}(2) \times \mathrm{U}(1)]_2$ is turned off, there is an $\mathrm{SU}(3)_2$ living in the lower-right 3×3 block. In this way it is guaranteed that the Higgs doublet does not receive one-loop quadratic divergences from the gauge boson loops.

The effective lagrangian of the model is written as

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_\psi + \mathcal{L}_t + \mathcal{L}_y\,, \tag{3.7}$$

where $\mathcal{L}_K$ contains the kinetic terms for the gauge and Goldstone bosons; $\mathcal{L}_\psi$ includes the kinetic terms for the fermions; $\mathcal{L}_t$ generates the Yukawa coupling for the top quark; and $\mathcal{L}_y$ generates the Yukawa couplings for the light quarks.

The gauge and scalar kinetic terms $\mathcal{L}_K$ of the littlest Higgs model is invariant under the T-parity which is defined as

$$\begin{aligned} \Pi &\leftrightarrow -\Omega\,\Pi\,\Omega\,, \\ A_1 &\leftrightarrow A_2\,, \end{aligned} \tag{3.8}$$

where $\Omega \equiv \mathrm{diag}(1,1,-1,1,1)$ commutes with all the gauge generators and Σ_0. Similar to the previous section, Ω is included in the definition of the T-parity to flip the parity of the Higgs doublet. In this way, the light fields including the standard model gauge fields and the Higgs doublet are even under T-parity, while the heavy gauge bosons and the scalar triplet, which picks up a mass $\sim 1\,\mathrm{TeV}$ at one loop, are T-odd.

The kinetic term for the SU(2) singlet fermions can be written down as in eq. (2.8), then they only couple to the standard model gauge bosons. To write down a kinetic term for the SU(2) doublet fermion, we introduce the object $\xi = \exp(i\Pi/f)$, just like in the T-invariant minimal moose model. Since $\Sigma(x) = \xi^2\Sigma_0$, the transformation property of ξ under SU(5) can be deduced from that of $\Sigma(x) \to V\Sigma(x)V^T$:

$$\xi \to V\xi U^\dagger = U\xi(\Sigma_0 V^T \Sigma_0) \tag{3.9}$$

where $U = \exp(iu^a(\Pi, V)T^a)$ belongs to the unbroken SO(5) and is a non-linear representation of the SU(5); the function u^a depends on the Goldstone fields Π and the SU(5) rotation V in a non-linear way. For a set of fermions furnishing a complete multiplet of the unbroken SO(5), the kinetic term is written down as in eq. (2.15). Here we just need to know how to embed an electroweak doublet in an SO(5) representation. Since it is easy to embed an electroweak doublet in an SU(5) representation, we begin by observing that, in the basis we choose, a fundamental representation of SU(5) decomposes into two copies of fundamental representation of the unbroken SO(5):

$$\psi = \begin{pmatrix} \psi_1 \\ \chi \\ \psi_2 \end{pmatrix} \to \psi \pm \Sigma_0 \psi^* , \tag{3.10}$$

where ψ_1, ψ_2 are doublets and χ is a singlet under $\mathrm{SU}(2)_V$. Now we can simply put an electroweak doublet fermion ψ_D of the standard model in the position of ψ_1. However, without any additional fermions, the kinetic term breaks all global symmetries that protect the Higgs mass, and a two-loop quartic-divergent contribution to the Higgs mass-squared will be generated as discussed in the previous section. There are two ways to deal with this problem. The first is to enlarge the scalar sector and introduce additional electroweak doublet, just like in the moose model. For example, we can double the Goldstone bosons by having two Σ fields. Then there are two electroweak doublets. One doublet can get a TeV mass from the fermion kinetic terms by the two-loop quartic divergences, but the other can remain light and be the SM Higgs doublet. Other additional Goldstone bosons such as triplets and singlets can be lifted by either the one-loop contributions or appropriate (Ω-dependent) plaquette operators.

Alternatively, we can cancel the quartic divergences by introducing new vector-pair of singlet and doublet fermions, χ, χ^c and $\tilde{\psi}_D$, $\tilde{\psi}^c_D$, respectively, for each electroweak doublet fermion in the standard model. Put ψ_D, χ, $\tilde{\psi}_D$ into the complete multiplet of the SO(5):

$$\psi = \begin{pmatrix} \psi_D \\ \chi \\ \tilde{\psi}_D \end{pmatrix} \to U\psi \tag{3.11}$$

and introduce Dirac mass terms, $m_\chi \chi^c \chi$ and $m_{\tilde{\psi}_D} \tilde{\psi}_D \tilde{\psi}_D^c$, for the new fermions. The kinetic term is now written as

$$\overline{\psi}\,\bar{\sigma}^\mu \left(\partial_\mu + v_\mu^a T^a + i g_Y (Y_{\psi_D} - Y_H) B_\mu\right) \psi\,, \tag{3.12}$$

where v_μ^a is defined as in eq. (2.11). The kinetic term is invariant under the T-parity with ψ transforming as $\psi \to \Omega\psi$, which implies that χ, χ^c are T-odd and $\tilde{\psi}_D$, $\tilde{\psi}_D^c$ are T-even. The kinetic term does not contain any direct couplings between standard model fermions and the heavy gauge bosons. They are contained in the T-odd term

$$\overline{\psi}\,\bar{\sigma}^\mu p_\mu^a X^a \psi\,, \tag{3.13}$$

which is forbidden by T-parity.

There is still a question about how to embed the doublet $\tilde{\psi}_D^c$ which marries with $\tilde{\psi}_D$. In the low energy theory, a possibility is to embed $\tilde{\psi}_D^c$, χ^c, together with an additional singlet χ'^c into a spinor representation of SO(5). (χ'^c can get a mass $m_{\chi'} \chi'^c \chi'$ with another singlet χ'.) Then its kinetic term can be written down following CCWZ. Alternatively, one can extend the theory with a third SU(2) gauge group, which is neutral under the T-parity, and have $\tilde{\psi}_D^c$ be a doublet of this extra SU(2). This extra SU(2) is broken together with the two SU(2) gauge groups inside the SU(5) down to the diagonal SM SU(2). The Dirac mass term $m_{\tilde{\psi}_D} \tilde{\psi}_D \tilde{\psi}_D^c$ can then arise from the Yukawa type interactions after the breaking. The gauge coupling of the third SU(2) can be large so that the mass of the extra gauge bosons is near the cutoff, and the SM gauge fields are mostly made of the gauge fields of the two SU(2)'s inside the SU(5). This is in the similar spirit as the three-site moose model of ref. [28]. In fact, such extensions also allow us to construct UV extensions of the T-invariant SU(5)/ SO(5) model, which contain only linear representations, i.e., no CCWZ construction is needed [33]. More generally, a little Higgs model based on a symmetric space G/H can be made T-invariant by extending it to $(G \times H)/H$. In this way, all fermions can be assigned to linear representations under the full group $G \times H$ without invoking CCWZ.

It can be explicitly checked that, in the presence of χ and $\tilde{\psi}_D$, the quartic divergence in figure 1 is cancelled by diagrams involving χ and $\tilde{\psi}_D$ similar to those in figure 2. Because we have the complete fermion multiplet, the only symmetry breaking sources in the kinetic term come from the gauge generators $Q_{1,2}^a$. We already know that each gauge generator preserves either $\mathrm{SU}(3)_1$ or $\mathrm{SU}(3)_2$ in the global SU(5) group, so for each interaction

$$\overline{\psi}\,\xi^\dagger\, Q_{1,2}^a\, \xi\, \psi \qquad \text{and} \qquad \overline{\psi}\,\xi\, Q_{1,2}^a\, \xi^\dagger\, \psi\,, \tag{3.14}$$

there is an unbroken SU(3) symmetry protecting the Higgs doublet from getting a mass. A Higgs mass can be induced only in the presence of both Q_1^a and Q_2^a, which requires going to higher loops or a mass insertion between A_1^a and A_2^a. Either way the contribution to the Higgs mass-squared can be no bigger than the two loop *quadratic* divergences and hence is safe. Because the quartic divergence in figure 1 is cut off by the masses of the extra fermions, m_χ, $m_{\tilde{\psi}_D}$, for naturalness they need to be below $\sim 4\pi\sqrt{m_h f/g}$.

Next step is to write down a top Yukawa coupling without introducing the associated quadratically divergent contribution to the Higgs mass, which can be achieved by adding

JHEP08(2004)061

the usual pair of T-even colored fermions t' and t'^c, as well as three additional colored, fermionic weak singlet fermions, u^c, u^c_T, and U. Grouping the third generation quark doublet $q_3 = (b\,t)$ with t' and $\tilde{q}_3$ into a row vector $\tilde{Q}_3^T = (q_3\; t'\; \tilde{q}_3)$, the top Yukawa coupling $\mathcal{L}_t$ can be written as[4]

$$\begin{aligned}\mathcal{L}_t = &\frac{1}{4\sqrt{2}}\lambda_1 f \epsilon_{ijk}\epsilon_{xy}\left[(\xi\tilde{Q}_3)_i(\xi^2\Sigma_0)_{jx}(\xi^2\Sigma_0)_{ky}u^c + (\tilde{\xi}\tilde{Q}_3)_i(\tilde{\xi}^2\Sigma_0)_{jx}(\tilde{\xi}^2\Sigma_0)_{ky}u^c_T\right] + \\ &+\lambda_2 f t' t'^c + \frac{\lambda_3 f}{\sqrt{2}}U(u^c - u^c_T) + \text{h.c.}\,, \end{aligned} \tag{3.15}$$

where $i,j,k = 1,2,3$, $x,y = 4,5$, and $\tilde{\xi} = \Omega\xi^\dagger\Omega$. This is nothing but the T-symmetrized version of the top Yukawa interaction written down in ref. [12], which can be easily recognized by noting that $\xi^2\Sigma_0 = \Sigma(x)$. We define u^c_T to be the image of u^c under T-parity, so that the first term in the square bracket in eq. (3.15) is mapped into the second under the T-parity. The singlet fermion U is taken to be T-odd and marries the T-odd linear combination $U^c \equiv (u^c - u^c_T)/\sqrt{2}$. Any T-odd interactions have been projected out in eq. (3.15). One way to show that there are no one-loop quadratic divergences generated from eq. (3.15) is to perform a spurion analysis as in ref. [11]. For that purpose we can regard $\mathcal{L}_t$ as having four independent interactions with coupling constants $\lambda_1, \lambda_1', \lambda_2$ and λ_3, where λ_1' is the coefficient of $(\xi\tilde{Q}_3)_i(\xi^2\Sigma_0)_{jx}(\xi^2\Sigma_0)_{ky}u^c_T$ and forced by T-parity to be equal to λ_1. Each interaction in $\mathcal{L}_t$ preserves enough SU(3) global symmetries to individually protect the Higgs mass from getting one-loop quadratic divergences; it takes more than one λ to get a quadratically divergent mass. Moreover, with the introduction of new fermions, each λ has a global U(1) rephasing symmetry associated with redefining the phases of u^c, u^c_T, t'^c and U. Therefore these spurions can only enter as $|\lambda^{(\prime)}/4\pi|^2$ and the quadratic divergence does not come in until two-loop order. Without the rephasing symmetries, there could be divergences proportional to, for instance, $(\lambda_1/4\pi)(\lambda_1'/4\pi)$ which would be too big a contribution to the Higgs mass-squared.

Expanding $\mathcal{L}_t$ to leading order in the Higgs particle gives

$$\lambda_1(\sqrt{2}H^\dagger q_3 + f t')u^c_3 + \lambda_2 f t' t'^c + \cdots\,, \tag{3.16}$$

where $u^c_3 = (u^c + u^c_T)/\sqrt{2}$ is the T-even linear combination of u^c and u^c_T. As usual, a linear combination $T^c \approx (\lambda_1 u^c_3 + \lambda_2 t'^c)/\sqrt{\lambda_1^2 + \lambda_2^2}$ marries with $T \approx t'$ (plus a small mixture from t) and becomes heavy with a mass $M_T \approx \sqrt{\lambda_1^2+\lambda_2^2}f$. After integrating out the heavy top partner T, the orthogonal combination t^c has the desired Yukawa coupling to q_3 with the strength $\lambda_t = \sqrt{2}\lambda_1\lambda_2/\sqrt{\lambda_1^2+\lambda_2^2}$.

4. Little Higgs phenomenology with T-parity

Little Higgs models without T-parity are in general strongly constrained by the precision electroweak measurements. The leading corrections to the electroweak observables in these

[4] We thank J.R. Espinosa for pointing out the Yukawa coupling in an early version of this paper leads to unacceptably large Higgs mass.

models come from several sources. First, the heavy gauge bosons mix with the SM gauge bosons through vertices like $H^\dagger W^\mu W'_\mu H$ after electroweak symmetry breaking, when the Higgs doublet gets a VEV. This shifts the masses of W and Z and also modifies the couplings of W and Z to the SM fermions. Secondly, integrating out heavy gauge bosons gives additional contributions to 4-fermion interactions at low energies if the heavy gauge bosons couple to SM fermions directly. In particular, the contribution to the Fermi constant G_F from muon decays feeds into all precision measurements since G_F is used as an input parameter. Finally, for models containing scalar triplets, a VEV of the triplet $\langle\phi\rangle$ will be induced by the Higgs VEV if there is a coupling $H\phi H$. The triplet VEV also shifts the W boson mass. These corrections are all of the order $(\langle H\rangle/f)^2$. Because many electroweak observables such as W mass and Z couplings to fermions are very accurately measured, the scale of f is in general constrained to be somewhat larger than 1 TeV as most phenomenological studies on little Higgs theories found. This in turn re-introduces fine-tuning in the Higgs mass. For example, in the original littlest Higgs model, the SU(5)/ SO(5) model without T-parity, the scale f needs to be above 4 TeV or so [19, 20, 21] in order to be compatible with precision data.

On the other hand, with T-parity all the above tree-level corrections to the electroweak observables are absent because they violate the T-parity. Apart from the unknown contributions from the cutoff physics, the corrections to the electroweak observables within the effective theory are loop-suppressed. One example of such constraints is the correction to $\Delta\rho$ parameter due to the loop of the heavy top quark partner, which is responsible for cancelling the quadratic divergence from the top Yukawa coupling. This correction is in the order of $(\langle H\rangle/f)^2/16\pi^2$ and very small. The correction to the $Z\bar{b}b$ vertex also gives comparable effect. In addition, the heavy gauge bosons could also contribute to the ρ parameter at one loop level. All these constraints are down by a loop factor and allow the scale f to be well below 1 TeV.

T-parity also drastically affects the collider phenomenology of the little Higgs theories. The T-odd particles cannot be singly produced due to T-parity, which implies that direct searches must rely on pair-productions. When the T-odd particles are produced, they will eventually decay into the LTP, the lightest T-odd particle, which is stable since T-parity forbids it from decaying into lighter particles which are all T-even. A charged LTP is not preferred as it can cause cosmological problems. In T-invariant models the LTP is most likely to be the B' gauge boson because its mass is suppressed by the smallness of the $\mathrm{U}(1)_Y$ coupling and also often by the normalization factor of the U(1) generator, although some other neutral scalar is also possible. For a neutral LTP, the typical collider signals will be the T-odd particles decaying into jets and/or leptons plus missing energies. In this respect the phenomenology is similar to those of the supersymmetric theories with R-parity and KK-parity conserving universal extra-dimensions (UED) [34, 35, 36], where the lightest supersymmetric particle (LSP) and the lightest KK-odd particle (LKP) are also stable and would escape detection in collider experiments if they are neutral. Moreover, the LTP could serve as a viable dark matter candidate, similar to LSP and LKP. One distinct feature of the T-invariant little Higgs theories is that the top sector still contains a T-even heavy top quark partner, whose phenomenology is not affected by the T-parity.

It can be used to test the little Higgs mechanism in colliders, as shown in ref. [37]. The T-even top partner can be singly produced and its decay products do not necessarily give missing energies. A recent study show that it can be observed up to mass of order 2.5 TeV via its decay to Wb at the LHC [38]. On the other hand, for the T-odd particles such as heavy gauge bosons and scalar triplets, all the previous phenomenological studies without assuming the T-parity do not apply. The details of their phenomenology depend on the particle content and the spectrum of each individual model. In the rest of this section, we will describe in a little more detail the phenomenology of the T-invariant SU(5)/ SO(5) model as an example.

Compared with the original littlest Higgs model, the T-invariant model has an extended scalar sector or fermion sector (or both) in the TeV scale. Here we consider the model with an extended fermion sector. Then, in the scalar and gauge sectors, the T-invariant SU(5)/ SO(5) model has the same matter/field content as the littlest Higgs model. There is one Higgs doublet H which is light, one heavy triplet scalar ϕ, and a set of massive $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge bosons W' and B'. The triplet scalar, responsible for cancelling the quadratic divergence of the Higgs quartic coupling, cannot get a VEV here because it is T-odd and forbidden to have a tadpole term from coupling to the Higgs doublet after electroweak symmetry breaking. The gauge couplings of the two $\mathrm{SU}(2) \times \mathrm{U}(1)$ groups are required to be equal by T-parity, implying $g_1 = g_2 = \sqrt{2}g$ and $g'_1 = g'_2 = \sqrt{2}g_Y$, where g and g_Y are the gauge coupling constants for the electroweak $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$. The mixing angles in the gauge couplings are set by the T-parity to be $\pi/4$, instead of being free parameters as in the littlest Higgs model. The masses for various gauge bosons are given by

$$\begin{aligned}
M_W^2 &= \frac{1}{2}g^2 f^2 \sin^2 \frac{\langle H \rangle}{f}\,, \\
M_Z^2 &= \frac{1}{2}(g^2 + g_Y^2) f^2 \sin^2 \frac{\langle H \rangle}{f}\,, \\
M_{W'}^2 &= g^2 f^2 - \frac{1}{2}g^2 f^2 \sin^2 \frac{\langle H \rangle}{f}\,, \\
\begin{pmatrix} M_{B'}^2 & M_{B'W'_3}^2 \\ M_{B'W'_3}^2 & M_{W'_3}^2 \end{pmatrix} &= \begin{pmatrix} \frac{1}{5}g_Y^2 f^2 - \frac{1}{8}g_Y^2 f^2 \sin^2 \frac{2\langle H \rangle}{f} & \frac{1}{8}g g_Y f^2 \sin^2 \frac{2\langle H \rangle}{f} \\ \frac{1}{8}g g_Y f^2 \sin^2 \frac{2\langle H \rangle}{f} & g^2 f^2 - \frac{1}{8}g^2 f^2 \sin^2 \frac{2\langle H \rangle}{f} \end{pmatrix}.
\end{aligned} \tag{4.1}$$

In the limit $f \gg \langle H \rangle$, these formulae reduce to the familiar ones:

$$M_W^2 \approx \frac{1}{4}g^2 v^2\,, \qquad M_Z^2 \approx \frac{1}{4}(g^2 + g_Y^2) v^2\,, \qquad M_{W'}^2 \approx g^2 f^2\,, \qquad M_{B'}^2 \approx \frac{1}{5}g_Y^2 f^2\,. \tag{4.2}$$

In the fermion sector, in addition to the top partners which cancels the quadratic divergence from the top Yukawa coupling, for every SM doublet fermion there is also a vector-like T-odd $\mathrm{SU}(2)_W$ singlet fermion and a vector-like T-even doublet fermion. If the masses of these fermions are not degenerate among different generations or aligned with the SM fermion masses, they can induce flavor-changing effects, such as $K^0 - \overline{K}^0$ mixing, $\mu \to e\gamma$ and so on. However, their couplings to SM fermions always involve the Higgs or the scalar triplet, e.g., $(1/f)\, \bar{\chi}\bar{\sigma}^\mu \mathcal{A}_\mu H^T \psi_{\mathrm{SM}}$, $(1/f^2)\, \bar{\tilde{\psi}}\bar{\sigma}^\mu A_\mu H H^T \psi_{\mathrm{SM}}$, $\bar{\tilde{\psi}}\bar{\sigma}^\mu \mathcal{A}_\mu \phi\, \psi_{\mathrm{SM}}$. Their contributions to the flavor-changing neutral currents are therefore suppressed either by powers of $\langle H\rangle/f$, or by higher loops if the physical Higgs boson or the scalar triplet is involved. For f, $m_{\chi,\tilde{\psi}} \sim 1\,\text{TeV}$, the induced flavor-changing effects are already safe even without degeneracies or alignments.

As we discussed earlier in this section, within the effective theory the corrections to the electroweak observables are loop-suppressed. The strongest constraint comes from the $\Delta\rho$ correction due to heavy gauge boson loops. One can see from eq. (4.1) that even if g_Y is turned off, the heavy $W'^{\pm}$ and W_3' gauge bosons are not degenerate and the mass splitting is

$$\Delta M'^2 = M^2_{W_3'} - M^2_{W'^{\pm}} = \frac{1}{2} g^2 f^2 \sin^4 \frac{\langle H\rangle}{f}\,. \tag{4.3}$$

This mass splitting induces at one-loop level a custodial $\mathrm{SU}(2)_C$ breaking effect and the correction to $\Delta\rho$ is given by

$$\Delta\rho_{W'} = -\frac{e^2}{s_W^2 c_W^2 M_Z^2}\left(\frac{9g^2}{64\pi^2}\Delta M'^2 \log\frac{\Lambda^2}{M^2_{W'}}\right), \tag{4.4}$$

where $s_W^2 = 1 - c_W^2 = \sin^2\theta_W$, and θ_W is the Weinberg angle. The logarithmic divergence means that a counter term in the non-linear sigma model of the form

$$\frac{g^2}{16\pi^2}\beta f^2 \sum_{i,a} \mathrm{Tr}[(Q_i^a D_\mu \Sigma)(Q_i^a D^\mu \Sigma)^*] \tag{4.5}$$

is required to cancel the divergence, where the numerical coefficient β is expected to be of order unity. Indeed, after turning on the Higgs VEV, this interaction gives a mass-splitting within the standard model SU(2) gauge bosons

$$\Delta M^2 = M^2_{W_3} - M^2_{W^{\pm}} = \beta\frac{g^4 f^2}{64\pi^2} \sin^4\frac{\langle H\rangle}{f}\,. \tag{4.6}$$

This is due to the fact that gauging two SU(2)'s in this model is a custodial $\mathrm{SU}(2)_C$ violating effect. There are further $\mathrm{SU}(2)_C$ breaking effects from the U(1) gauge couplings, but they are expected to be smaller due to the smallness of g_Y. Assuming there is no large counter term at the cutoff, the contribution to $\Delta\rho$ from W' loop between the M_W' and the cutoff Λ is negative and enhanced by a $\log(\Lambda/M_{W'})$ factor,

$$\Delta\rho_{W'} \approx -0.002\left(\frac{450\,\text{GeV}}{f}\right)^2\left(1 + 0.34\log\frac{\Lambda}{4\pi f}\right). \tag{4.7}$$

If one requires that $\Delta\rho > -0.002$, this only constrains the symmetry breaking scale f to be greater than 450 GeV which corresponds to

$$M_{W',Z'} \gtrsim 280\,\text{GeV}\,, \qquad M_{\hat{B}'} \gtrsim 60\,\text{GeV}\,, \qquad M_T \gtrsim 640\,\text{GeV}\,, \tag{4.8}$$

where $\hat{B}'$, Z' represent the mass eigenstates and are very close to the gauge eigenstates B', W_3'.

JHEP08(2004)061

The $\Delta\rho$ parameter also receives contribution from the top partner at the one-loop level,

$$\Delta\rho_{t'} \approx \frac{3\lambda_t^2}{16\pi^2}\sin^2\theta_t\left[\ln\left(\frac{M_T^2}{m_t^2}\right) - 1 + \frac{1}{2}\left(\frac{\lambda_1}{\lambda_2}\right)^2\right], \tag{4.9}$$

where

$$\sin^2\theta_t \approx \left(\frac{\lambda_1}{\lambda_2}\right)^2 \frac{m_t^2}{M_T^2} \tag{4.10}$$

is the mixing angle between the top quark and the top partner in the left-handed sector, and $M_T \approx \sqrt{\lambda_1^2+\lambda_2^2}f$ is the mass of the heavy top partner. The top Yukawa coupling is given by $\lambda_t \approx \sqrt{2}\lambda_1\lambda_2/\sqrt{\lambda_1^2+\lambda_2^2}$. It does not provide any significant constraint as long as λ_1/λ_2 is somewhat small ($\lesssim 0.8$). The correction to the $Zb\bar{b}$ vertex from the top partner loop has the same leading logarithm as in eq. (4.9) and gives comparable constraint to $\Delta\rho_{t'}$. The contributions to the S parameter of Peskin-Takeuchi [39, 40] are also small. In fact, the top partner contribution to $\Delta\rho$ is positive so it is possible to partially cancel the negative contribution from the W' loop, allowing f to be even lower. (The direct search limit is $m_{W'} \gtrsim 100\,\mathrm{GeV}$ from LEP II which corresponds to $f \approx 200\,\mathrm{GeV}$.) Of course, a very low f undermines the purpose of the little Higgs theories and one expects that the higher dimensional operators at the cutoff $\Lambda = 4\pi f$ will give too big contributions to the electroweak observables. Indeed, as the constraints from the calculable effects within the effective theory are rather weak in this model, the most stringent constraints are expected to come from the higher dimensional operators arising from unknown UV physics at the cutoff scale, which would prefer $\Lambda \approx 4\pi f \gtrsim 5-10\,\mathrm{TeV}$ if these unknown contributions are not suppressed.

In terms of collider phenomenology, the T-invariant model escapes conclusions reached by most studies in the literature so far, except for those on the T-even top partners (T, T^c). The T-odd particles in the model are the heavy gauge bosons, a triplet scalar, and the singlet fermions χ_{q_i}, χ_{l_i}, $i = 1, 2, 3$ and (U, U^c); they are produced in pairs through the SM gauge bosons and eventually decay into the LTP. It can be seen from eq. (4.2) that the gauge bosons are likely to be lighter than the triplet scalar and the heavy fermions. The relatively light B' boson is most likely to be the LTP, as will be assumed in the following studies. Moreover, electroweak symmetry breaking induces corrections to the mass of the heavy gauge bosons, as well as a small mixing between W_3' and B', which causes the mass eigenstate Z' to be slightly heavier than the W'. The mass splitting can be calculated from eq. (4.1) and is of the order of $\langle H\rangle^4/f^2$. The triplet scalar ϕ can be slightly heavier than 1 TeV, since it is only responsible for cancelling the quadratic divergence from the Higgs quartic coupling. The singlet fermions χ and doublet fermions $\tilde{\psi}$ can be 3 TeV or higher, because they are there only to cancel the quartic divergence from the fermion kinetic term. The additional fermions in the top sector, on the other hand, is at around 1 TeV or so.

Due to the T-parity, all the T-odd particles need to decay into another T-odd particle plus something, and will eventually cascade down to the LTP, resulting in the missing energy in the detector. W' is likely to be the next-to-lightest T-odd particle. It decays into W plus $\hat{B}'$ through the small W_3' component in $\hat{B}'$ due to the mixing. Z' decays into $W\,W\,\hat{B}'$

Particle	T-parity	Major decay channels
W'	$-$	$W\,\hat{B}'$
Z'	$-$	$W\,W\,\hat{B}'$
ϕ	$-$	$W(Z,h)\,W'(Z',\hat{B}')$
t'	$+$	$t\,h,\ t\,Z,\ b\,W$
U	$-$	$t\,\phi$
χ	$-$	$\psi_{\rm SM}\,W'(Z',\hat{B}')$
$\tilde{\psi}$	$+$	$\chi\,W'(Z',\hat{B}'),\ \psi_{\rm SM}\,W(Z)$

Table 1: The major decay channels of the heavy particles in the T-invariant SU(5)/ SO(5) model. The B' is assumed to be the LTP. The particles produced in the decays may be replaced by the ones in the parentheses.

through $W\,W'^*$ (one of them needs to be virtual because of small mass splitting between W' and Z'). Other heavy particles can decay through the interactions, $\phi^\dagger\mathcal{A}^\mu D_\mu(H\,H^T)$, $\bar{\chi}\bar{\sigma}^\mu\mathcal{A}_\mu H^T\psi_{\rm SM}$, $\bar{\tilde{\psi}}\bar{\sigma}^\mu\mathcal{A}_\mu H\chi$, $\bar{\tilde{\psi}}\gamma^\mu A_\mu HH^T\psi_{\rm SM}$, $\bar{\tilde{\psi}}\gamma^\mu\mathcal{A}_\mu\phi\,\psi_{\rm SM}$, and $H^\dagger\phi q_3U^c$, where H can be replaced by its VEV or the physical Higgs boson.

In table 1 we list the major decay modes for all the heavy particles in the T-invariant model. For concreteness we have assumed the following spectrum,

$$M_{\hat{B}'} < M_{W'} < M_\phi < m_T \lesssim m_U < m_\chi \lesssim m_{\tilde{\psi}}\,. \tag{4.11}$$

The decays of the lighter (and hence more accessible) states (W', Z', ϕ) often produce W, Z bosons associated with the missing energy, which could be the first things to look for in collider experiments, though separating them from SM backgrounds would be a challenge.

5. Conclusions

With Tevatron Run II currently running and LHC to start in 2007, the TeV scale physics will be fully explored in the coming decade. The mystery of the origin of the electroweak symmetry breaking is expected to be unveiled. So far the precision electroweak measurements have not provided any evidence for new physics beyond the standard model, which is quite puzzling. Perhaps the fact that there is no sign of new physics in the precision electroweak data itself is one of the biggest hints on the possible new physics that will show up at the TeV scale. In this paper, we show that the little Higgs theories, supplemented with the T-parity, can be a solution to this little hierarchy problem. The large quadratically divergent corrections to the Higgs mass-squared from the standard model particles are cancelled by those from new TeV particles, which stabilizes the electroweak scale. At the same time the T-parity forbids the tree-level contributions to the electroweak observables from the TeV particles, and makes the little Higgs theories consistent with the electroweak precision data without fine-tuning.

A subtlety for imposing T-parity on little Higgs theories is how to incorporate the standard model fermions. However, here we show that by following CCWZ, which is the most general way to construct the low energy effective theory for a broken symmetry, this

JHEP08(2004)061

problem is readily resolved. The essential T-even interactions and the troublesome T-odd interactions can be naturally separated, which allows for an obvious implementation of T-parity. Although we only discussed two examples, the minimal moose model and the littlest Higgs model, from the discussion it should be apparent that the T-parity can be introduced in many other models. As long as a model is based on a symmetric space with the unbroken and broken generators satisfying (3.2), the automorphism $T^a \to T^a$, $X^a \to -X^a$, allows us to define a T-parity consistently. For some models one needs to worry about the 2-loop quartic-divergent contributions to the Higgs mass-squared, but it can be easily addressed by completing fermions into complete multiplets of the unbroken group H or enlarging the global symmetry.

There are also little Higgs models which do not live in a symmetric coset space. One example is the little Higgs model based on a "simple group" [15]. In the simplest version (which all other variants based on), an SU(3) gauge symmetry is broken down to SU(2) by the VEVs of two triplets. In this case, the broken generator T^8 does not satisfy (3.2), and the coset space is not a symmetric space. One cannot find a consistent definition of T-parity under which all heavy gauge bosons are odd. Nevertheless, one can still follow the approach of CCWZ. The model-independent couplings of the SM fermions to the SM gauge bosons and the model-dependent couplings of the SM fermions to the heavy gauge bosons are still naturally separated into different terms. The difference is that in this case there is no symmetry to forbid the couplings to the heavy gauge bosons. In fact, they will be radiatively generated even if one does not include them in the first place. For such models, the electroweak precision constraints turn into constraints on these model dependent couplings. How much fine-tuning, if any, is required depends on the model and needs to be examined individually.

The existence of an exact T-parity obviously has a big impact on the little Higgs phenomenology. The T-odd particles need to be pair-produced and the lightest T-odd particle is stable. A neutral LTP provides a viable dark matter candidate and gives rise to missing energy signals in collider experiments. To a first approximation, the collider signals for this class of models are similar to those of the supersymmetric standard model. The details of what can be observed at the colliders depend on each individual model and its spectrum. They certainly deserve more thorough investigations. In order to distinguish the little Higgs theories with T-parity from supersymmetry, the information on the spins of the new particles participating in the cancellation of quadratic divergences will be crucial. In this respect, a linear collider with a high enough energy could be very helpful.

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JHEP08(2004)061

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JHEP08(2004)061